Sinusoidal Application Problems

1) Phases of the Moon

The percent of the moon's surface (y) that is illuminated on day x of the year 1995, where $x = 303$ represents October 30, is given below.

x	303	311	319	326	333	341
y	.5	1.0	.5	0	.5	1.0

a) Create a scatter plot of the data on your calculator.

b) Find a sinusoidal function that fits the data and graph it with the scatter plot.

c) Define the values a, b, c, d in the context of the problem for $y = a\sin b(x - c) + d$.

d) Find the first day of the year 1995 where the moon was 70% illuminated.

2) Solstices

According to the Old Farmer's Almanac, the number of hours of sunlight in Boston on the day of the summer solstice (day 172) is 15.283 and the number of hours of sunlight on the day of the winter solstice (day 355) is 9.067.

a) Find a sinusoidal function that fits the data and graph it with the scatter plot.

b) Draw a graph of the function found in (a).

c) Use the function found in (a) to predict the number of hours of sunlight on April 1, the 91^{st} day of the year.

3) Temperature in Alaska

Based on years of weather data, the expected low temperature T (in degrees F) in Fairbanks, Alaska, is a function of the date (t), where t is in days and $t = 0$ corresponds to January 1^{st}. The temperature can be approximated by

$$T = 36\sin\frac{2\pi}{365}(t-101)+14$$

a) Sketch the graph of T for 0 to 365.

b) Predict when the coldest day of the year will occur.

4) Temperature in Denver

The data given in the table represents the average monthly temperatures in Denver, Colorado. Since the data represent average monthly temperatures collected over many years, the data will not vary much from year to year and so will essentially repeat each year. In other words, the data are periodic.

Month	**Average Monthly Temp.**
January, x = 1	29.7
February	33.4
March	39.0
April	48.2
May	57.2
June	66.9
July	73.5
August	71.4
September	62.3
October	51.4
November	39.0
December	31.0

a) Create a scatter plot on your calculator, where x = 1 represents January, x = 2 represents February, and so on.

b) Find a sinusoidal function that fits the data and graph it with the scatter plot.

5) Average Precipitation

The average monthly precipitation *P* (in inches) at South Lake Tahoe, California, is listed in the table below.

Month	**P**	**Month**	**P**	**Month**	**P**
Jan	6.1	May	1.2	Sept	0.5
Feb	5.4	June	0.6	Oct	2.8
March	3.9	July	0.3	Nov	3.1
April	2.2	Aug.	0.2	Dec	5.4

a) Create a scatter plot on your calculator, where x = 1 represents January, x = 2 represents February, and so on.

b) Find a sinusoidal function that approximates the average monthly precipitation and graph it with the scatter plot.

Sinusoid Applications

Name ______________________

Solve each of the following both graphically and analytically. Be sure to check and justify your solutions. Choose and define meaningful variables where necessary and use functional notation.

1. Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, **d**, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 sec, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft, and it completed a revolution every 10 seconds.

a) Sketch a **meaningful** graph of the situation and then write an equation for the graph.

b) How far above the surface was the point when Mark's watch read: 5 sec and 17 sec?

c) What is the first positive value of time at which the point was at the water's surface? At that time, was it going into or coming out of the water? Explain.

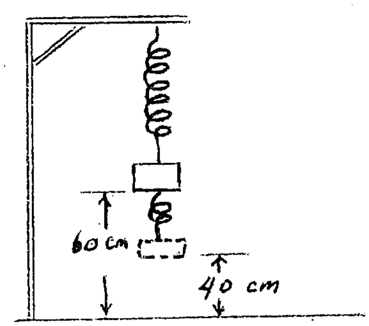
2. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 sec, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 sec.

a) Sketch a **meaningful** graph of the situation and write the equation of the sinusoid.

b) Predict the distance from the floor when the stopwatch reads 17.2 sec.

c) What was the distance from the floor when you started the stopwatch?

d) Predict the first positive value of time at which the weight is 59 cm above the floor.



3. Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let **t** be the number of seconds the stopwatch reads and let **y** be the number of meters Tarzan is from the riverbank. Assume that **y** varies sinusoidally with **t**, and that **y** us positive when Tarzan is over water and negative when he is over land.

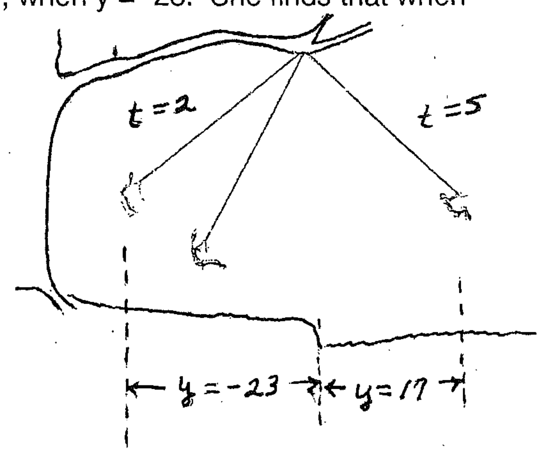
Jane finds that when t = 2, Tarzan is at one end of his swing, when y = -23. She finds that when t = 5 he reaches the other end of his swing and y = 17.

a) Sketch a **meaningful** graph and write an equation to model the situation.

b) Predict **y** when: t = 2.8 t = 6.3 t = 15

c) Where was Tărzan when Jane started the watch?

d) Find the least positive value of **t** for which Tarzan is directly over the riverbank.



Graphing Circular Functions

Sketch a graph of each function and include the 5 key x-values along with the 3 key y-values for each graph.

1) $y = 2\sin\pi x$

2) $y = 4\cos 1.5x$

3) $y = -2 + 0.5\cos\frac{\pi}{4}x$

4) $y = 3\sec\frac{\pi}{2}x$

5) $y = -4\csc(0.5x)$

6) $y = 3\tan(\frac{\pi x}{2} + \pi)$

7) $y = -5\sin\frac{x}{3}$

8) $y = 10\cos(\frac{x}{4} + \frac{\pi}{2})$

9) $y = 3 - 4\sin(2.5x + \pi)$

10) $y = 2 - \sec[\pi(x - 3)]$